MATH 512, SPRING 17
HOMEWORK 1, DUE WED FEBRUARY 1

Problem 1. *A measure U on κ is normal iff the diagonal function $d(\alpha) = \alpha$ for $\alpha < \kappa$ is the least function f (with respect to $<_U$) with the property that for every $\beta < \kappa$, $\{\alpha \mid f(\alpha) > \beta\} \in U$*

Problem 2. *Suppose D is a normal measure on κ and $f : [\kappa]^{<\omega} \to \kappa$ is such that for all x, $f(x) = 0$ or $f(x) < \min x$. Then there is $H \in D$ homogeneous for f, i.e. for all n, f is constant on $[H]^n$.*

Problem 3.
1. *Show that $\{\alpha < \omega_2 \mid \mathrm{cf}(\alpha) = \omega\}$ reflects.*
2. *Show that $\{\alpha < \omega_2 \mid \mathrm{cf}(\alpha) = \omega_1\}$ does not reflect. In general, show that for any regular cardinal κ, $\{\alpha < \kappa^+ \mid \mathrm{cf}(\alpha) = \kappa\}$ does not reflect.*

Problem 4. *The principle diamond $\diamond$ states that there is a sequence $\langle A_\alpha \mid \alpha < \omega_1\rangle$, such that each $A_\alpha \subset \alpha$ and for every set $A \subset \omega_1$, the set $\{\alpha < \omega_1 \mid A \cap \alpha = A_\alpha\}$ is stationary. Show that $\diamond$ implies CH.*

Recall that a cardinal κ is weakly compact iff κ is Π^1_1-indescribable.

Problem 5. *Suppose κ is weakly compact. Show that $Refl(\kappa)$ holds.*

Problem 6. *Suppose κ is weakly compact. Show that κ is Mahlo*

Problem 7. *Suppose that U is a nonprincipal κ-complete ultrafilter on κ. Let $M = Ult(U, V)$ and $j = j_U$ be the canonical elementary embedding of V to M. Show that:*
1. *$U \notin M$.*
2. *$\mathcal{P}(\kappa) = \mathcal{P}^M(\kappa)$ and $|j(\kappa)|^V = 2^\kappa$.*
3. *If λ is singular with $\mathrm{cf}(\lambda) = \kappa$, then $j(\lambda) > \sup_{\alpha<\lambda} j(\alpha)$.*
4. *If λ is singular with $\mathrm{cf}(\lambda) \neq \kappa$, then $j(\lambda) = \sup_{\alpha<\lambda} j(\alpha)$.*
5. *If λ is a strong limit cardinal, with $\mathrm{cf}(\lambda) \neq \kappa$, then $j(\lambda) = \lambda$.*

Problem 8. *If κ is measurable and GCH holds below κ (i.e. $2^\tau = \tau^+$ for every $\tau < \kappa$), then it holds at κ i.e. $2^\kappa = \kappa^+$.*

Problem 9. *Let $\kappa < \lambda$ be uncountable cardinals.*
1. *Show that $A = \{x \in \mathcal{P}_\kappa(\lambda) \mid \kappa \cap x \in \kappa\}$ is club in $\mathcal{P}_\kappa(\lambda)$. For every $x \in A$, denote $\kappa_x := x \cap \kappa$.*
2. *Suppose that C is a club in $\mathcal{P}_\kappa(\lambda)$, such that $C \subset A$, where A is the set above. Show that $\{\kappa_x \mid x \in C\}$ is club in κ.*

Problem 10. *Let $\kappa < \lambda$ be uncountable cardinals, and suppose that κ is λ-supercompact. Show that κ is measurable.*